AP Calculus (BC) Chapter 6 Test
No Calculator Section

Name:_____________________________ **Date:** ________________ **Period:** _______

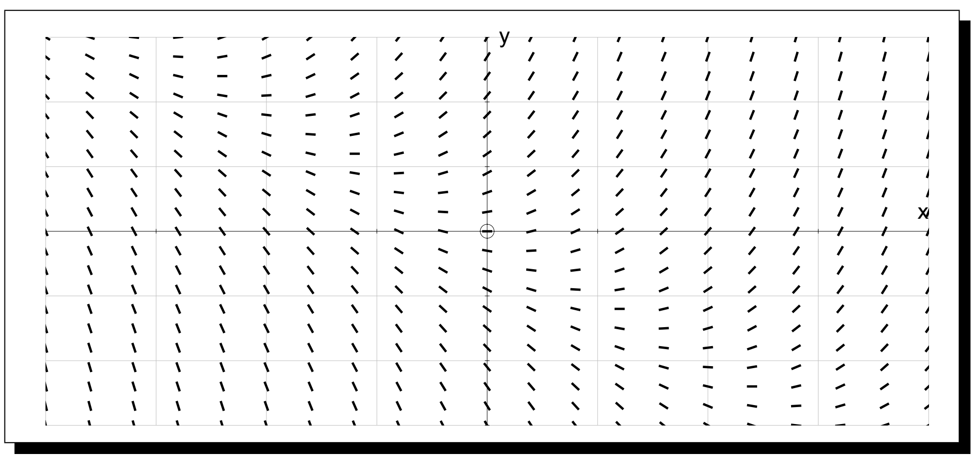
Part I. Multiple-Choice Questions (5 points each; please circle the correct answer.)

1. $\int_0^{\sqrt{\pi/2}} x \cos x^2 \, dx \; =$

   (A) 1
   (B) -1
   (C) $\frac{1}{2}$
   (D) $-\frac{1}{2}$
   (E) 0

2. The slope field indicated below most likely depicts solutions of the differential equation

   (A) $\frac{dy}{dx} = x + y$
   (B) $\frac{dy}{dx} = x - y$
   (C) $\frac{dy}{dx} = x + y^2$
   (D) $\frac{dy}{dx} = x^2 + y$
   (E) $\frac{dy}{dx} = xy$



3. $\displaystyle\int_0^{\pi/2} x \sin 2x \, dx =$

(A) 0
(B) $\dfrac{\pi}{4}$
(C) $-\dfrac{\pi}{4}$
(D) $\dfrac{\pi}{2}$
(E) $-\dfrac{\pi}{2}$

4. Given the **logistic differential equation** $\dfrac{dA}{dt} = A\left(20 - \dfrac{A}{4}\right)$, where $A(0) = 15$, what is $\lim\limits_{t\to\infty} A(t)$?

(A) 20
(B) 40
(C) 60
(D) 80
(E) 100

5. Given the differential equation $\dfrac{dT}{dt} = -\dfrac{1}{4}(T - 20),\ T(0) = 100$, then $T(20) =$

(A) $80e^{-5}$
(B) $100e^{-5}$
(C) $80e^{-5} + 20$
(D) $80e^{-5} - 20$
(E) 20

Part II. Free-Response Questions

6. (A) **(5 points)** Evaluate the indefinite integral $\int x^2 \cos 2x \, dx$.

   (B) **(4 points)** Using your result in (A), compute $\int_0^{\pi/4} x^2 \cos 2x \, dx$.

7. Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.

(A) **(8 points)** Let $y = f(x)$ be the particular solution to the given differential equation for $1 \leq x \leq 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

(B) **(8 points)** Compute $y = f(x)$ explicitly, given the above information.

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Part III. Multiple-Choice Questions (5 points each; please circle the correct answer.)

8. You are given the differential equation $\frac{dy}{dx} = -\frac{1}{5}\sqrt{y}$, where $y(0) = 25$. For which value(s) of x is $y = 0$?

(A) ± 5
(B) 10
(C) 50
(D) 10 and -10
(E) 5

9. $\displaystyle\int_{-\pi}^{\pi} \frac{\cos x\,dx}{\sqrt{4+3\sin x}} =$

(A) $\frac{4}{3}$
(B) $-\frac{4}{3}$
(C) 0
(D) $\frac{4}{3}\sqrt{7}$
(E) $-\frac{4}{3}\sqrt{7}$

10. $\displaystyle\int_{0}^{1} \sin^{-1} x\,dx =$

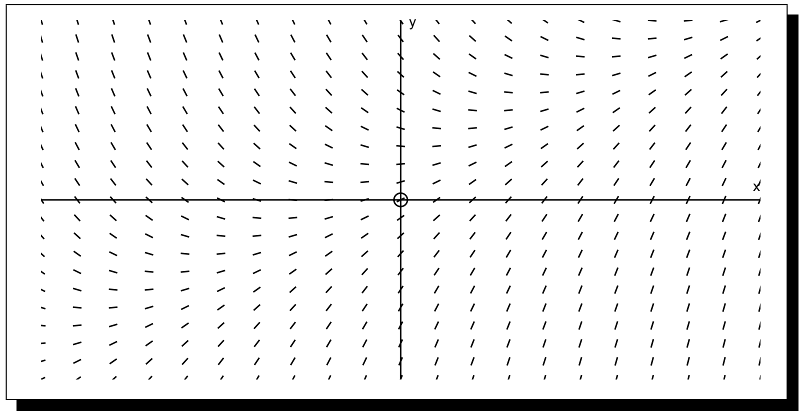
(A) 0
(B) $\frac{\pi+2}{2}$
(C) $\frac{\pi-2}{2}$
(D) $\frac{\pi}{2}$
(E) $-\frac{\pi}{2}$

11. The initial-value problem $\frac{dy}{dx} = f(x, y),\ y(0) = y_0 > 0$ has its slope field represented below. Which of the following are true?

I. The graph of the solution is concave up for all x.

II. $\lim_{x \to -\infty} y = -\infty$.

III. The graph of the solution has a slant asymptote.



(A) I only

(B) II only

(C) III only

(D) I and II

(E) I and III

12. If s is the displacement of a particle, then $v = \frac{ds}{dt}$ is its velocity. Assume that we are given that the particle has positive mass m and that $m\frac{dv}{dt} = -kv,\ v(0) > 0$, where k is a **positive** constant. Which of the following are true?

I. $\lim_{t \to \infty} s = \infty$.

II. $\lim_{t \to \infty} v = 0$.

III. The graph of v as a function of t is concave up.

(A) I only

(B) II only

(C) I and II

(D) II and III

(E) I and III

Part IV. Free-Response Questions

13. Suppose that the spread of measles in our high school is predicted by the logistic differential equation

$$\frac{dP}{dt} = \frac{k}{650}(650 - P)P, \quad P(0) = 1,$$

where t is the number of days after a student comes into contact with an infected student. We have seen that this has solution given by

$$P(t) = \frac{650}{1 + Ce^{-kt}}, \quad t \geq 0,$$

and where $C,\ k$ are positive constants.

(A) **(2 points)** Compute C.

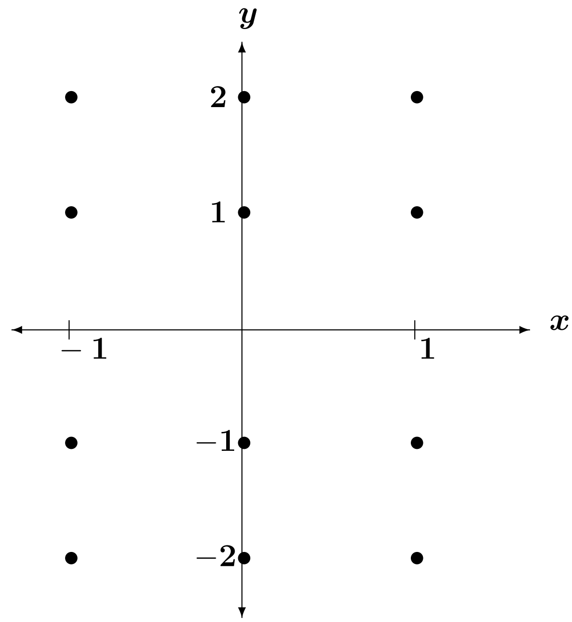
(B) **(5 points)** Assuming that the maximum rate of infections is 10 students/day, find k.

(C) **(3 points)** After how many days will half of the students have become infected?

14. Consider the differential equation

$$\frac{dy}{dx} = -\frac{2x}{y}.$$

(A) **(4 points)** On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(B) **(6 points)** Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

(c) **(5 points)** Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.